# Reachability in VAS is not Elementary

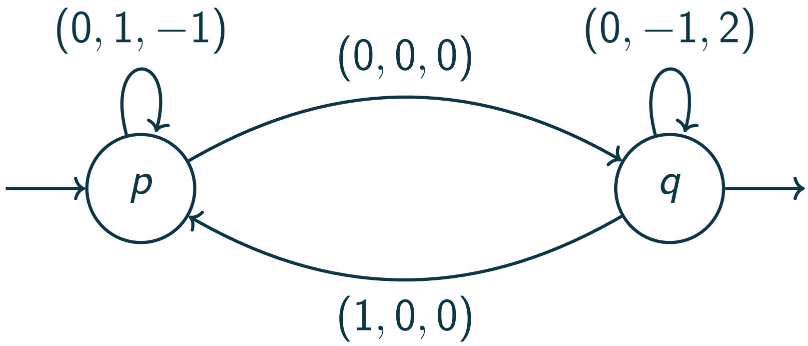
Jérôme Leroux (LaBRI : CNRS & Univ. Bordeaux & Bordeaux-INP)
Joint work with W. Czerwiński, S. Lasota, R. Lazić and F. Mazowiecki.

# Reachability in VAS is not Elementary

Jérôme Leroux (LaBRI : CNRS & Univ. Bordeaux & Bordeaux-INP)
Joint work with W. Czerwiński, S. Lasota, R. Lazić and F. Mazowiecki.

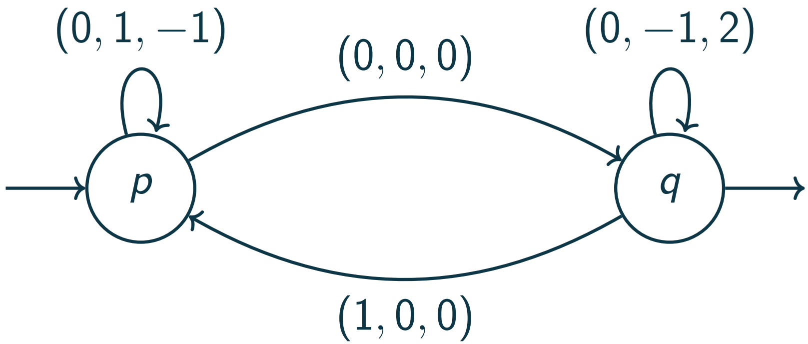
# Vector addition systems with states (VASS)



# Vector addition systems with states (VASS)

States

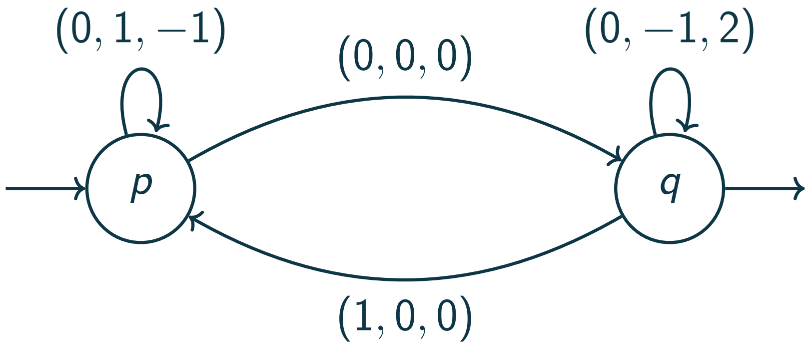
$p, q$



# Vector addition systems with states (VASS)

States

$p, q$



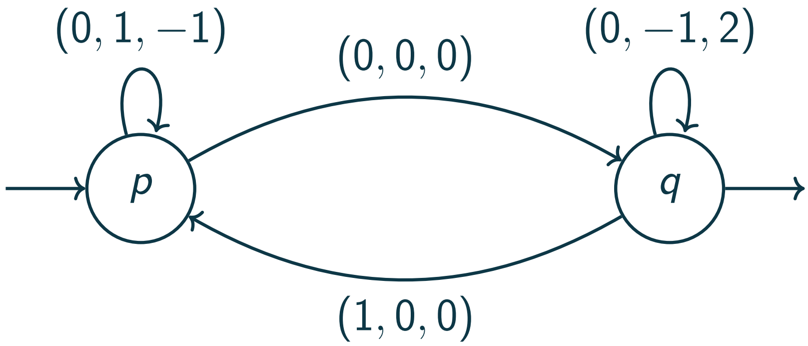

Actions

$(0,1,-1)$, $(0,0,0)$, $(1,0,0)$, $(0,-1,2)$

# Vector addition systems with states (VASS)

States

$p$, $q$



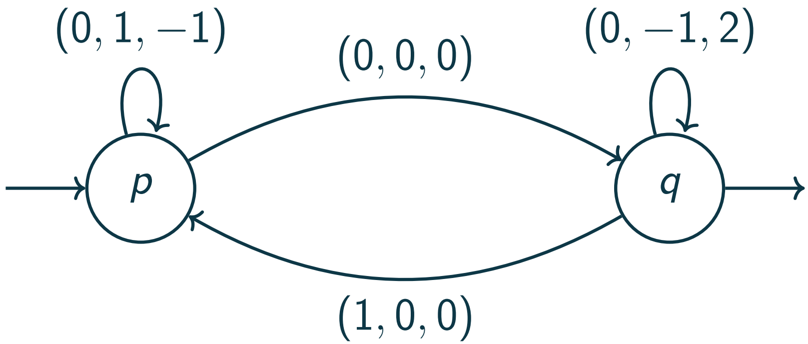

Actions

$(0,1,-1)$, $(0,0,0)$, $(1,0,0)$, $(0,-1,2)$

Dimension

3

# Vector addition systems with states (VASS)




States

$p,\ q$

Actions

$(0,1,-1),\ (0,0,0),\ (1,0,0),\ (0,-1,2)$

Dimension

3

---

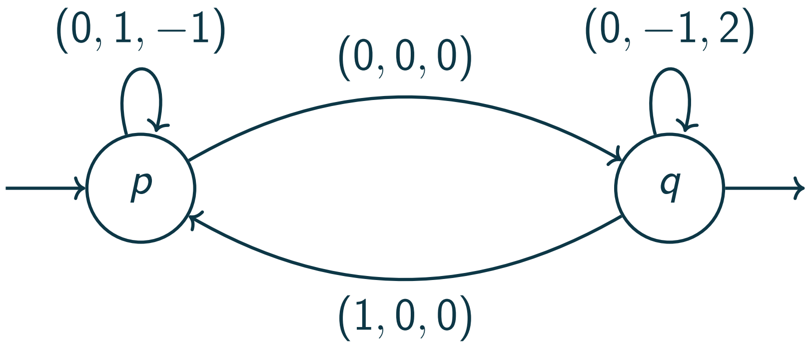
Configurations

$p(x,y,z),\ q(x,y,z)$ with $x,y,z \in \mathbb{N}$

# Vector addition systems with states (VASS)

States

$p, \; q$




Actions

$(0,1,-1),\ (0,0,0),\ (1,0,0),\ (0,-1,2)$

Dimension

3

---

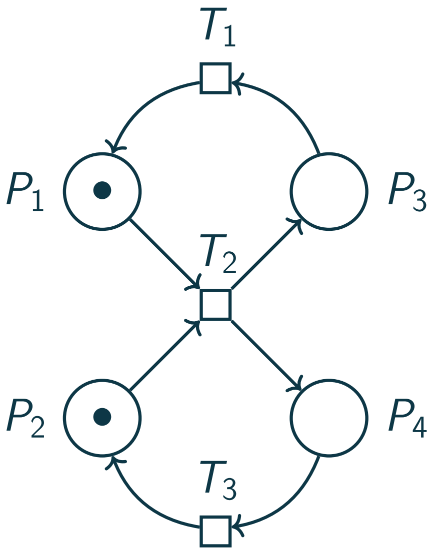
Configurations

$p(x, y, z),\ q(x, y, z)$ with $x, y, z \in \mathbb{N}$

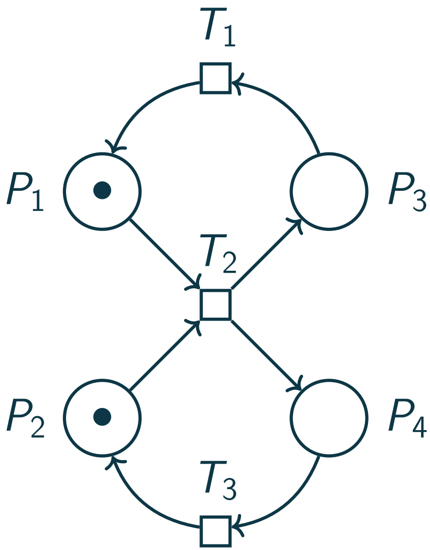
Runs

$p(0,0,1) \to p(0,1,0) \to q(0,1,0)$ also written $p(0,0,1) \xrightarrow{*} q(0,1,0)$

# Petri nets



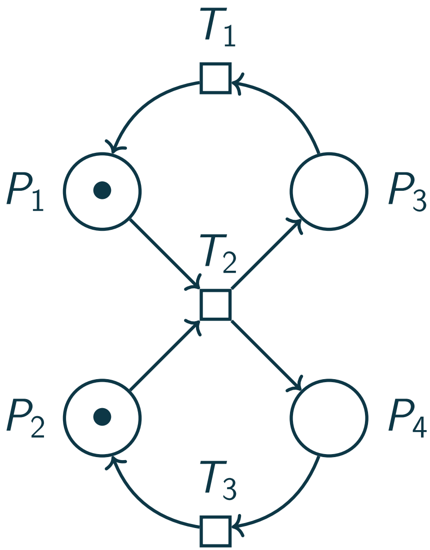
# Petri nets

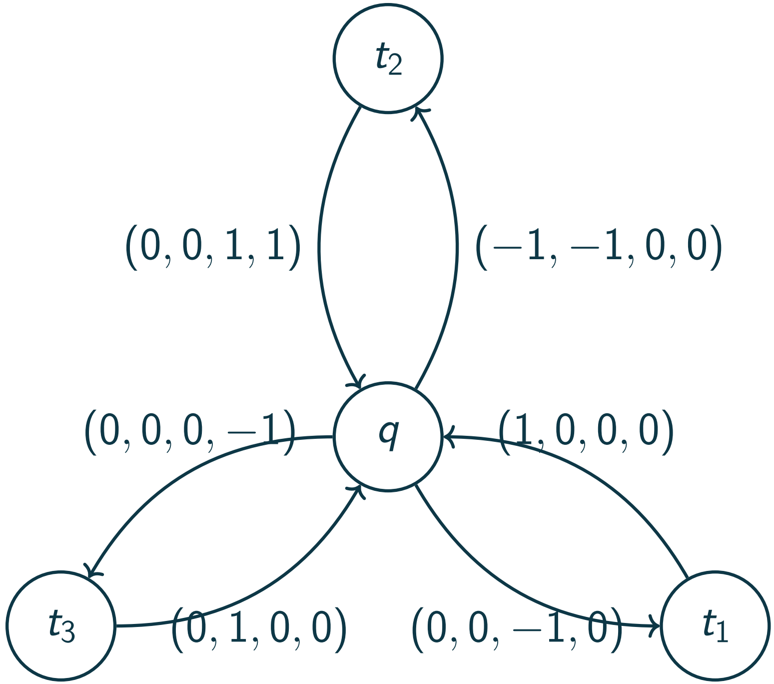



Model of concurrency with extensive applications in modelling and analysis of

- hardware and software,
- database systems,
- chemical, biological and business processes.

# Petri nets




Petri nets $\longleftrightarrow$ VASS

## Decision problems

Reachability problem:

GIVEN: a VASS $V$

DECIDE: whether $p(0, \ldots, 0) \xrightarrow{*} q(0, \ldots, 0)$ with $p$ initial and $q$ final ?

# Decision problems

Reachability problem:

GIVEN: a VASS $V$

DECIDE: whether $p(0, \ldots, 0) \xrightarrow{*} q(0, \ldots, 0)$ with $p$ initial and $q$ final ?

Coverability problem:

GIVEN: a VASS $V$

DECIDE: whether exists v s.t. $p(0, \ldots, 0) \xrightarrow{*} q(\mathrm{v})$ with $p$ initial and $q$ final ?

# Decision problems

Reachability problem:

GIVEN: a VASS $V$

DECIDE: whether $p(0,\ldots,0) \xrightarrow{*} q(0,\ldots,0)$ with $p$ initial and $q$ final ?

Coverability problem:

GIVEN: a VASS $V$

DECIDE: whether exists v s.t. $p(0,\ldots,0) \xrightarrow{*} q(\mathrm{v})$ with $p$ initial and $q$ final ?

Many problems reduce to reachability or coverability

- Formal languages
- Logic
- Concurrent systems
- Process calculi,...

# Reachability state of the art

# Reachability state of the art

1976 — Expspace lower bound (Lipton)

## Reachability state of the art

1976 — Expspace lower bound (Lipton)

1981 — Decidable (Mayr)

## Reachability state of the art

1976 — Expspace lower bound (Lipton)

1981 — Decidable (Mayr)

1982 — Decidable (Kosaraju)

# Reachability state of the art

| | |
|---|---|
| 1976 | Expspace lower bound (Lipton) |
| 1981 | Decidable (Mayr) |
| 1982 | Decidable (Kosaraju) |
| 1992 | Decidable (Lambert) |

# Reachability state of the art

| | |
|---|---|
| 1976 | Expspace lower bound (Lipton) |
| 1981 | Decidable (Mayr) |
| 1982 | Decidable (Kosaraju) |
| 1992 | Decidable (Lambert) |
| 2009-2012 | Decidable with Presburger inductive invariants (Leroux) |

# Reachability state of the art

1976 — Expspace lower bound (Lipton)

1981 — Decidable (Mayr)

1982 — Decidable (Kosaraju)

1992 — Decidable (Lambert)

2009-2012 — Decidable with Presburger inductive invariants (Leroux)

2015-2019 — $F_\omega$ upper bound (Leroux and Schmitz)

# Reachability state of the art

1976 — Expspace lower bound (Lipton)

1981 — Decidable (Mayr)

1982 — Decidable (Kosaraju)

1992 — Decidable (Lambert)

2009-2012 — Decidable with Presburger inductive invariants (Leroux)

2015-2019 — $F_\omega$ upper bound (Leroux and Schmitz)

2019 — $F_3$ lower bound (Czerwiński, Lasota, Lazić, Leroux and Mazowiecki)

# Reachability state of the art

- 1976 — Expspace lower bound (Lipton)
- Coverability Rackoff 1978 Exspace-complete
- 1981 — Decidable (Mayr)
- 1982 — Decidable (Kosaraju)
- 1992 — Decidable (Lambert)
- 2009-2012 — Decidable with Presburger inductive invariants (Leroux)
- 2015-2019 — $F_\omega$ upper bound (Leroux and Schmitz)
- 2019 — $F_3$ lower bound (Czerwiński, Lasota, Lazić, Leroux and Mazowiecki)

# Outline

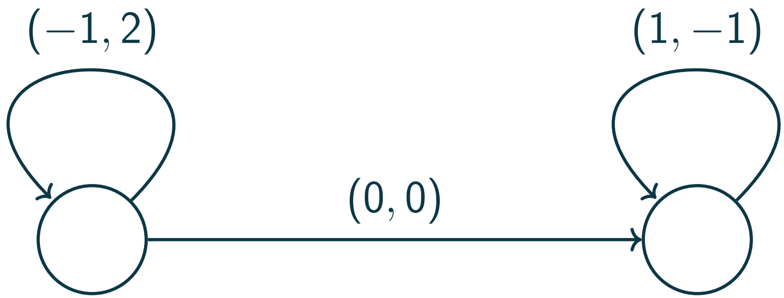
**1.** Large Configurations

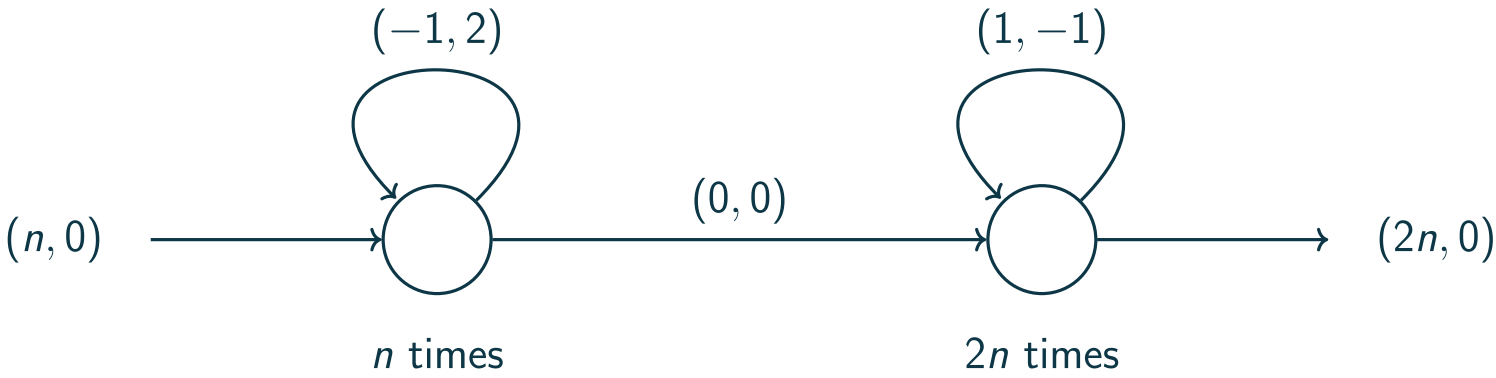
**2.** Witness of Unreachability

**3.** Counter Programs

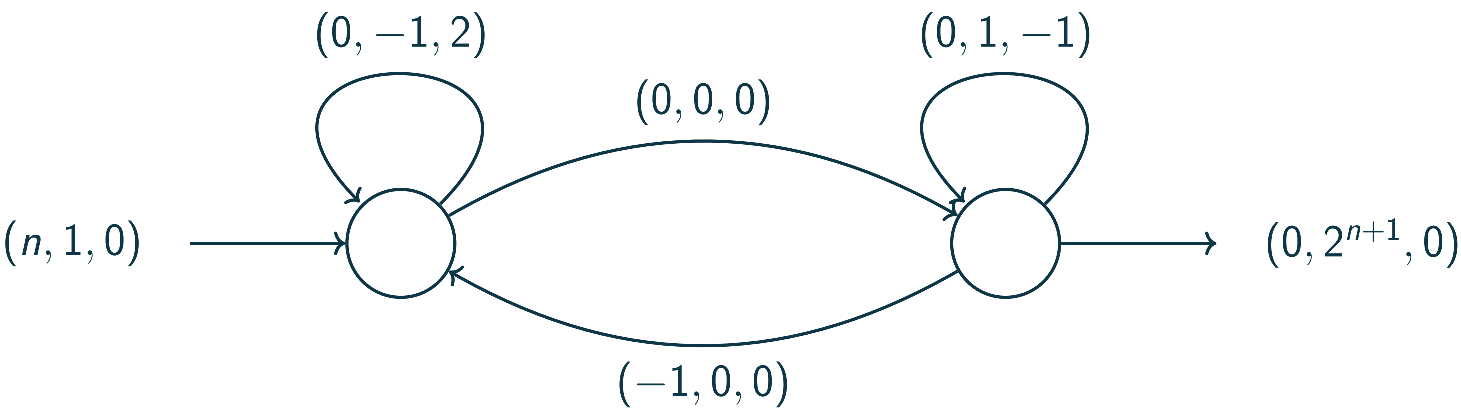
**4.** Lower Bounds

# Large Configurations


(−1, 2)
(1, −1)
(0, 0)


(−1, 2)
(1, −1)
(n, 0)
(0, 0)
(2n, 0)
n times
2n times

# The Hopcroft-Pansiot 1979 Example




Exponential behavior

# Fast Growing Functions

$$F_0(n) = n + 1$$

$$F_{d+1}(n) = F_d^{n+1}(n)$$

# Fast Growing Functions

$$F_0(n) = n + 1$$

$$F_{d+1}(n) = F_d^{n+1}(n)$$

$$F_1(n) = 2n + 1$$

# Fast Growing Functions

$$F_0(n) = n + 1$$

$$F_{d+1}(n) = F_d^{n+1}(n)$$

$$F_1(n) = 2n + 1$$

$$F_2(n) = 2^{n+1}(n + 1) - 1$$

# Fast Growing Functions

$$F_0(n) = n + 1$$

$$F_{d+1}(n) = F_d^{n+1}(n)$$

$$F_1(n) = 2n + 1$$

$$F_2(n) = 2^{n+1}(n + 1) - 1$$

$$F_3(n) \sim Tower(n)$$

...

# Fast Growing Functions

$$F_0(n) = n + 1$$

$$F_{d+1}(n) = F_d^{n+1}(n)$$

$$F_1(n) = 2n + 1$$

$$F_2(n) = 2^{n+1}(n + 1) - 1$$

$$F_3(n) \sim Tower(n)$$

...

$F_d$ weakly computable with VASSes in dim $d + 1$ (Mayr and Meyer 1981).

# Outline

1. Large Configurations
2. Witness of Unreachability
3. Counter Programs
4. Lower Bounds

# Witness of Unreachability

# Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

# Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

$2\mathbb{N} \qquad \phi(x) := \exists k \; x = k + k$

## Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

$2\mathbb{N} \qquad \phi(x) := \exists k\ x = k + k$

$\{0\} \qquad \phi(x) := x = x + x$

# Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

- $2\mathbb{N}$ $\quad \phi(x) := \exists k\ x = k + k$
- $\{0\}$ $\quad \phi(x) := x = x + x$
- $\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\}$ $\quad \phi(x, y) := \exists k\ y = x + k$

# Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

$2\mathbb{N} \qquad \phi(x) := \exists k \; x = k + k$

$\{0\} \qquad \phi(x) := x = x + x$

$\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\} \qquad \phi(x, y) := \exists k \; y = x + k$

$\{1\} \qquad \phi(x) := \forall k \; k \leq x \wedge k \neq x \Rightarrow k = 0$

# Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

$2\mathbb{N} \quad \phi(x) := \exists k\ x = k + k$

$\{0\} \quad \phi(x) := x = x + x$

$\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\} \quad \phi(x, y) := \exists k\ y = x + k$

$\{1\} \quad \phi(x) := \forall k\ k \leq x \wedge k \neq x \Rightarrow k = 0$

**Theorem** (Ginsburg and Spanier 1966)

A subset of $\mathbb{N}^d$ is Presburger (definable) if, and only if, it is *semilinear*.

I.e. a finite union of sets of the form $\vec{b} + \mathbb{N}\vec{p}_1 + \cdots + \mathbb{N}\vec{p}_k$

# Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

$2\mathbb{N} \qquad \phi(x) := \exists k \; x = k + k$

$\{0\} \qquad \phi(x) := x = x + x$

$\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\} \qquad \phi(x, y) := \exists k \; y = x + k$

$\{1\} \qquad \phi(x) := \forall k \; k \leq x \wedge k \neq x \Rightarrow k = 0$

**Theorem** (Ginsburg and Spanier 1966)

A subset of $\mathbb{N}^d$ is Presburger (definable) if, and only if, it is *semilinear*.

I.e. a finite union of sets of the form $\vec{b} + \mathbb{N}\vec{p}_1 + \cdots + \mathbb{N}\vec{p}_k$

$2\mathbb{N} \qquad 0 + \mathbb{N}2$

## Presburger Arithmetic

$\mathrm{FO}(\mathbb{N}, +)$

$2\mathbb{N}$ $\quad \phi(x) := \exists k \; x = k + k$

$\{0\}$ $\quad \phi(x) := x = x + x$

$\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\}$ $\quad \phi(x, y) := \exists k \; y = x + k$

$\{1\}$ $\quad \phi(x) := \forall k \; k \leq x \wedge k \neq x \Rightarrow k = 0$

**Theorem** (Ginsburg and Spanier 1966)

A subset of $\mathbb{N}^d$ is Presburger (definable) if, and only if, it is *semilinear*.

I.e. a finite union of sets of the form $\vec{b} + \mathbb{N}\vec{p}_1 + \cdots + \mathbb{N}\vec{p}_k$

$2\mathbb{N}$ $\quad 0 + \mathbb{N}2$

$\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\}$ $\quad (0, 0) + \mathbb{N}(0, 1)$

# Presburger Witness of Unreachability

# Presburger Witness of Unreachability

A **witness of unreachability** $C$ for a VASS is a set of configurations such that:

- $q(0,\ldots,0) \in C$ for every initial state $q$
- $c \to c' \wedge c \in C \Rightarrow c' \in C$
- $q(0,\ldots,0) \notin C$ for every final state $q$

# Presburger Witness of Unreachability

A **witness of unreachability** $C$ for a VASS is a set of configurations such that:

- $q(0,\ldots,0) \in C$ for every initial state $q$
- $c \to c' \wedge c \in C \Rightarrow c' \in C$
- $q(0,\ldots,0) \notin C$ for every final state $q$

**Theorem** (Leroux 2009-2012)

For every VASS:

- Either $p(0,\ldots,0) \xrightarrow{*} q(0,\ldots,0)$ with $p$ initial and $q$ final
- Or there exists a Presburger witness of unreachability

# Outline

**1.** Large Configurations

**2.** Witness of Unreachability

**3.** Counter Programs

**4.** Lower Bounds

# Counter Programs

# Counter Programs

# Counter Programs

- Operations over bounded counters $\bar{\mathsf{x}} \in \{0, \ldots, B\}$:

  $\bar{\mathsf{x}} \mathrel{+}= 1$

  $\bar{\mathsf{x}} \mathrel{-}= 1$

  **zero?** $\bar{\mathsf{x}}$

  **max?** $\bar{\mathsf{x}}$

- Operations over unbounded counters $\mathsf{x} \in \mathbb{N}$:

  $\mathsf{x} \mathrel{+}= 1$

  $\mathsf{x} \mathrel{-}= 1$

- Non deterministic **loop**

# Examples

$$\bar{a} \mathrel{+}= 2 \longrightarrow \begin{array}{l} \bar{a} \mathrel{+}= 1 \\ \bar{a} \mathrel{+}= 1 \end{array} \qquad x \mathrel{+}= 2 \longrightarrow \begin{array}{l} x \mathrel{+}= 1 \\ x \mathrel{+}= 1 \end{array}$$

# Examples

$$\bar{a} \mathrel{+}= 2 \quad\longrightarrow\quad \begin{array}{l} \bar{a} \mathrel{+}= 1 \\ \bar{a} \mathrel{+}= 1 \end{array} \qquad\qquad x \mathrel{+}= 2 \quad\longrightarrow\quad \begin{array}{l} x \mathrel{+}= 1 \\ x \mathrel{+}= 1 \end{array}$$

$$\textbf{assert } \bar{a} = 2 \quad\longrightarrow\quad \begin{array}{l} \bar{a} \mathrel{-}= 2 \\ \textbf{zero? } \bar{a} \\ \bar{a} \mathrel{+}= 2 \end{array}$$

# Examples

$$\bar{a} \mathrel{+}= 2 \quad \longrightarrow \quad \begin{array}{l} \bar{a} \mathrel{+}= 1 \\ \bar{a} \mathrel{+}= 1 \end{array} \qquad\qquad x \mathrel{+}= 2 \quad \longrightarrow \quad \begin{array}{l} x \mathrel{+}= 1 \\ x \mathrel{+}= 1 \end{array}$$

$$\textbf{assert } \bar{a} = 2 \quad \longrightarrow \quad \begin{array}{l} \bar{a} \mathrel{-}= 2 \\ \textbf{zero? } \bar{a} \\ \bar{a} \mathrel{+}= 2 \end{array}$$

$$\bar{a} := 2 \quad \longrightarrow \quad \begin{array}{l} \textbf{loop} \\ \qquad \bar{a} \mathrel{-}= 1 \\ \textbf{zero? } \bar{a} \\ \bar{a} \mathrel{+}= 2 \end{array}$$

## Reachability Problems

A run is **complete** if it starts and ends with zero in every counter.

# Reachability Problems

A run is **complete** if it starts and ends with zero in every counter.

Reachability problem (for counter programs):

GIVEN: A counter program and a bound $B$.

DECIDE: Does it have a complete run ?

# Reachability Problems

A run is **complete** if it starts and ends with zero in every counter.

<u>Reachability problem (for counter programs)</u>:

GIVEN: A counter program and a bound $B$.

DECIDE: Does it have a complete run ?

<u>Reachability problem (for VASS)</u>:

GIVEN: a VASS $V$.

DECIDE: whether $p(0,\ldots,0) \xrightarrow{*} q(0,\ldots,0)$ with $p$ initial and $q$ final ?

# Reachability Problems

A run is **complete** if it starts and ends with zero in every counter.

Reachability problem (for counter programs):

GIVEN: A counter program and a bound $B$.

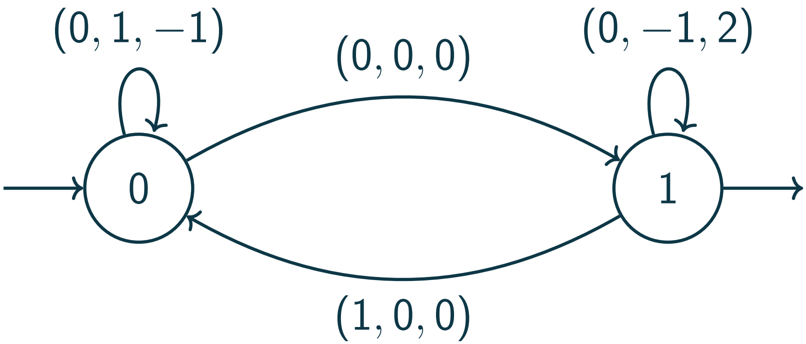
DECIDE: Does it have a complete run ?

Reachability problem (for VASS):

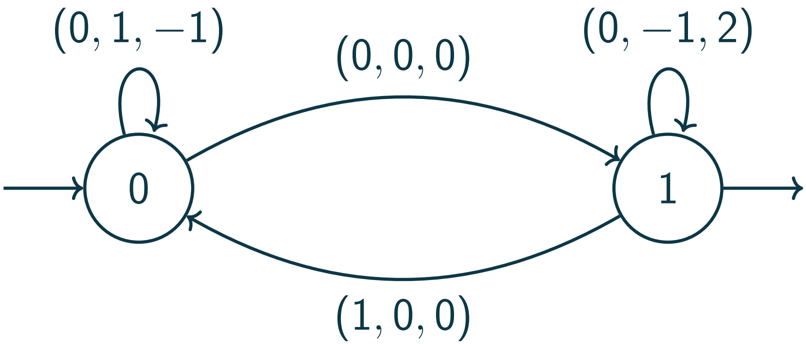
GIVEN: a VASS $V$.

DECIDE: whether $p(0,\ldots,0) \xrightarrow{*} q(0,\ldots,0)$ with $p$ initial and $q$ final ?

# VASS → Counter Programs



# VASS → Counter Programs




**loop**
    **loop**
        **assert** $\bar{a} = 0$
        $y \mathrel{+}= 1 \quad z \mathrel{-}= 1 \quad \bar{a} := 0$
    **loop**
        **assert** $\bar{a} = 0$
        $\bar{a} := 1$
    **loop**
        **assert** $\bar{a} = 1$
        $y \mathrel{-}= 1 \quad z \mathrel{+}= 2 \quad \bar{a} := 1$
    **loop**
        **assert** $\bar{a} = 1$
        $x \mathrel{+}= 1 \quad \bar{a} := 0$
**assert** $\bar{a} = 1$
$\bar{a} := 0$

with $B = 1$.

# Counter Programs → VASS

$\bar{\mathsf{i}} \mathrel{+}= 1$

**loop**

    $\mathsf{x} \mathrel{+}= 1$

    $\bar{\mathsf{i}} \mathrel{+}= 1$

$\bar{\mathsf{i}} \mathrel{-}= 1$

with $B = 2$.

# Counter Programs → VASS

1: $\bar{\mathsf{i}}$ += 1

2: **loop**

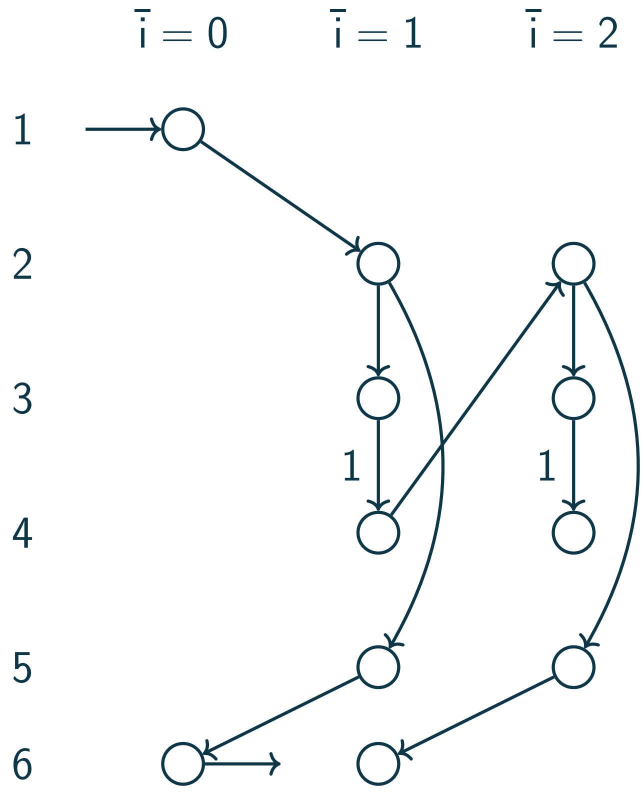
3: x += 1

4: $\bar{\mathsf{i}}$ += 1

5: $\bar{\mathsf{i}}$ −= 1

6:

with $B = 2$.



# Outline

1. Large Configurations
2. Witness of Unreachability
3. Counter Programs
4. Lower Bounds

# Lower Bounds

# Main idea

Implement with a counter program:

$$n \cdot \prod_{1 \leq i < B} \frac{i+1}{i} = n \cdot B$$

# A weak multiplier by $\frac{3}{2}$

**loop**

$\quad x \mathrel{-}= 2 \quad x' \mathrel{+}= 3$

**loop**

$\quad x' \mathrel{-}= 1 \quad x \mathrel{+}= 1$

# A weak multiplier by $\frac{3}{2}$

**loop**

$x \mathrel{-}= 2 \quad x' \mathrel{+}= 3$

**loop**

$x' \mathrel{-}= 1 \quad x \mathrel{+}= 1$

| $x$ | $x'$ |
|---|---|
| 14 | 0 |
| 12 | 3 |
| 10 | 6 |
| 8 | 9 |
| 6 | 12 |
| 4 | 15 |
| 2 | 18 |
| 0 | 21 |
| 1 | 20 |
| 2 | 19 |
| $\vdots$ | $\vdots$ |
| 20 | 1 |
| $\frac{3}{2}14 = 21$ | 0 |

# A weak multiplier by $\frac{3}{2}$

**loop**

$x \mathrel{-}= 2 \quad x' \mathrel{+}= 3$

**loop**

$x' \mathrel{-}= 1 \quad x \mathrel{+}= 1$

| $x$ | $x'$ |
|---|---|
| 14 | 0 |
| 12 | 3 |
| 10 | 6 |
| 8 | 9 |
| 6 | 12 |
| 4 | 15 |
| 2 | 18 |
| 0 | 21 |
| 1 | 20 |
| 2 | 19 |
| $\vdots$ | $\vdots$ |
| 20 | 1 |
| $\frac{3}{2}14 = 21$ | 0 |

| $x$ | $x'$ |
|---|---|
| 15 | 0 |
| 13 | 3 |
| 11 | 6 |
| 9 | 9 |
| 7 | 12 |
| 5 | 15 |
| 3 | 18 |
| 1 | 21 |
| 2 | 20 |
| 3 | 19 |
| $\vdots$ | $\vdots$ |
| 21 | 1 |
| $\frac{3}{2}15 > 22$ | 0 |

# A weak multiplier by $\frac{\bar{i}+1}{\bar{i}}$

**loop**

$\quad x \mathrel{-}= \bar{i} \qquad x' \mathrel{+}= \bar{i}+1$

**loop**

$\quad x' \mathrel{-}= 1 \qquad x \mathrel{+}= 1$

# A weak multiplier by $\frac{\bar{i}+1}{\bar{i}}$

**loop**

$\quad x \mathrel{-}= 1 \quad \bar{\mathsf{i}} \mathrel{-}= 1 \quad \bar{\mathsf{a}} \mathrel{+}= 1$

**zero?** $\bar{\mathsf{i}}$

**loop**

$\quad \bar{\mathsf{i}} \mathrel{+}= 1 \quad \bar{\mathsf{a}} \mathrel{-}= 1$

**zero?** $\bar{\mathsf{a}}$

**loop**

$\quad x \mathrel{-}= \bar{\mathsf{i}} \quad x' \mathrel{+}= \bar{\mathsf{i}} + 1$

**loop**

$\quad x' \mathrel{-}= 1 \quad x \mathrel{+}= 1$

# A weak multiplier by $\frac{\bar{i}+1}{\bar{i}}$

**loop**

  $x \mathrel{-}= \bar{i} \quad x' \mathrel{+}= \bar{i}+1$

**loop**

  $x' \mathrel{-}= 1 \quad x \mathrel{+}= 1$

**loop**

  $x \mathrel{-}= 1 \quad \bar{i} \mathrel{-}= 1 \quad \bar{a} \mathrel{+}= 1$

**zero?** $\bar{i}$

**loop**

  $\bar{i} \mathrel{+}= 1 \quad \bar{a} \mathrel{-}= 1$

**zero?** $\bar{a}$

$x' \mathrel{+}= 1$

**loop**

  $x' \mathrel{+}= 1 \quad \bar{i} \mathrel{-}= 1 \quad \bar{a} \mathrel{+}= 1$

**zero?** $\bar{i}$

**loop**

  $\bar{i} \mathrel{+}= 1 \quad \bar{a} \mathrel{-}= 1$

**zero?** $\bar{a}$

## Implementing $n \cdot \prod_{1 \leq i < B} \frac{i+1}{i} = n \cdot B$

## Implementing $n \cdot \prod_{1 \leq i < B} \frac{i+1}{i} = n \cdot B$

$$\left.\begin{array}{l} \mathsf{x} \mathrel{+}= 1 \quad \mathsf{y} \mathrel{+}= 1 \\ \textbf{loop} \\ \quad \mathsf{x} \mathrel{+}= 1 \quad \mathsf{y} \mathrel{+}= 1 \\ \bar{\mathsf{i}} \mathrel{+}= 1 \end{array}\right\} \text{init x and y to some } n \geq 1$$

**loop**

$$\left.\begin{array}{l} \textbf{loop} \\ \quad \mathsf{x} \mathrel{-}= \bar{\mathsf{i}} \quad \mathsf{x}' \mathrel{+}= \bar{\mathsf{i}} + 1 \\ \textbf{loop} \\ \quad \mathsf{x}' \mathrel{-}= 1 \quad \mathsf{x} \mathrel{+}= 1 \\ \bar{\mathsf{i}} \mathrel{+}= 1 \end{array}\right\} \text{weak multiplier by } \frac{\bar{\mathsf{i}}+1}{\bar{\mathsf{i}}}$$

**max?** $\bar{\mathsf{i}}$

$$\left.\begin{array}{l} \textbf{loop} \\ \quad \mathsf{x} \mathrel{-}= \bar{\mathsf{i}} \quad \mathsf{y} \mathrel{-}= 1 \end{array}\right\} \text{multiplication checker}$$

$$\left.\begin{array}{l} \textbf{loop} \\ \quad \bar{\mathsf{i}} \mathrel{-}= 1 \end{array}\right\} \text{reset } \bar{\mathsf{i}}$$

# Lower Bounds

Factorial amplifier:

- Simulate counters bounded by $B!$ with counters bounded by $B$.

# Lower Bounds

Factorial amplifier:

- Simulate counters bounded by $B!$ with counters bounded by $B$.

VASS reachability problem:

- $F_3$ lower bound.
- $d$-EXPSPACE lower bound in dim $d + 13$.

# Conclusion

- Reachability $>>>$ Coverability

## Conclusion

- Reachability $>>>$ Coverability

- Plethora of problems are not elementary

  In formal languages, logic, concurrent systems, process calculi,...

# Conclusion

- Reachability $>>>$ Coverability

- Plethora of problems are not elementary

  In formal languages, logic, concurrent systems, process calculi,...

- We can do $d$-EXPSPACE-hardness in dimension $d + 13$ (so fixed)

  Can we do Tower in fixed dimension?

# Conclusion

- Reachability $>>>$ Coverability
- Plethora of problems are not elementary

  In formal languages, logic, concurrent systems, process calculi,...
- We can do $d$-EXPSPACE-hardness in dimension $d + 13$ (so fixed)

  Can we do Tower in fixed dimension?
- The complexity is still open

  Between Tower ($F_3$) and Ackermann ($F_\omega$)

## Conclusion

- Reachability $>>>$ Coverability
- Plethora of problems are not elementary

  In formal languages, logic, concurrent systems, process calculi,...
- We can do $d$-EXPSPACE-hardness in dimension $d + 13$ (so fixed)

  Can we do Tower in fixed dimension?
- The complexity is still open

  Between Tower ($F_3$) and Ackermann ($F_\omega$)
- Up to 15 months postdoc position in Bordeaux and/or Cachan (France)